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Work on this program included four topics: General Laser Resonator Theory, Distributed Reflection of Light and Damage to Optical Elements, Oxide Layer Thickness on Commercial ALCLAD, and Multipass Chemical Laser Power Amplifiers. In all of these areas, fundamentally new approaches, techniques, and results were developed and achieved.

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FINAL SCIENTIFIC REPORT

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Grant for Basic Research

AFOSR-77-3278

ANALYSIS OF UNSTABLE OPTICAL RESONATORS

For the Period: March 1, 1977 to March 31, 1979

Submitted by
Dr. John D. Reichert
Optical Sciences Laboratory
Department of Electrical Engineering
Texas Tech University
Lubbock, Texas 79409

June 9, 1980

Texas Tech University
Optical Sciences Laboratory
Report Number:
TTOSL-FSR-2

John D. Reichert
Dr. John D. Reichert
Principal Investigator

ABSTRACT

The U. S. Air Force Grant for Basic Research AFOSR-77-3278 to the Texas Tech Optical Sciences Laboratory was in effect for the period March 1, 1977 to March 31, 1979. The work of AFOSR-77-3278 was a continuation of research initiated under Grant No. AFOSR-73-2451 which covered the period October 1, 1972 to February 28, 1977. The total period covered by these two grants covered six and one-half years. The work included four topics: General Laser Resonator Theory, Distributed Reflection of Light and Damage to Optical Elements, Oxide Layer Thickness on Commercial ALCLAD, and Multipass Chemical Laser Power Amplifiers. In all of these areas, fundamentally new approaches, techniques, and results were developed and achieved. The work was interrupted on March 31, 1979 because, instead of a formal proposal for continuation, Texas Tech submitted, instead, a request for Air Force guidance and evaluation of the relevance of the work to the Air Force Mission. This evaluation, presumably, is not yet complete, but it is hoped that the work will be continued in the future.

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FINAL SCIENTIFIC REPORT

AFOSR-77-3278

The U. S. Air Force Grant AFOSR-77-3278 was in effect for the period March 1, 1977 to March 31, 1979. The work was a continuation of work initiated under the grant AFOSR-73-2451 which covered the period October 1, 1972 to February 28, 1977. Over the period of these two grants, comprising six and one-half years, work was conducted on four topics: General Laser Resonator Theory (GRT), Distributed Reflection of Light and Damage to Optical Elements (DRL), Oxide Layer Thickness on Commercial ALCLAD (OLT), and Multipass Chemical Laser Power Amplifiers (MPA). In all of these areas, fundamentally new approaches, techniques and results were achieved.

Eight reports, comprising 555 pages, were completed and submitted under AFOSR-73-2451. The Final Summary Report, TTOSL-FSR-1, for AFOSR-73-2451, filed September 12, 1977, reviewed this initial work. Two additional reports, comprising 282 pages, were completed and submitted under AFOSR-77-3278. These reports are listed in the REPORTS section below. In Section II, a brief overview is given for the work on General Laser Resonator Theory accomplished under this grant. Specifically, a mathematically consistent integral equation developed under the AFOSR funding to Texas Tech, the Source Integral Equation, is compared with the Fox-Li integral equation and the nature of the true laser resonance condition is exposed. The analysis is presented in terms of a functional equation method, developed under the AFOSR grants, which is quite useful for approximate solution of either integral equation. The Fox-Li equation does not account properly for the transverse dimensions of the cavity in the resonance condition. The mathematical structure of the actual laser resonance condition, revealed for the first time under our AFOSR grants, is illustrated. Possible future work is mentioned in Section III.

I. REPORTS

Two reports, comprising 282 pages, were completed and submitted under AFOSR-77-3278. These are listed below. For description of eight previous reports, comprising 555 pages, see the Final Summary Report, TTOSL-FSR-1, for AFOSR-73-2451, filed September 12, 1977.

TTOSL-OLT-3, August 1, 1977, (137 pages)

> "Ellipsometric Determination of Properties of Films on
> Rough Surfaces such as Aluminum Alloy Aircraft Skin" by
> Dr. John D. Reichert and
> Janet S. Brock

TTOSL-GRT-2, December 1, 1977, (145 pages)

> "Integral Equations and Functional Methods for Laser
> Mode Profiles" by
> Dr. John D. Reichert and
> Dr. Ajit Pal Kwatra

II. OVERVIEW OF GENERAL RESONATOR THEORY TOPIC

Based upon experience with somewhat similar boundary value problems, one might expect a different type of laser resonance condition than that provided by the Fox-Li equation and other "pseudo initial value problem" approaches. It is indeed the case that the role of the transverse resonator dimensions is altered in character and not fully delineated by such approaches. This and related matters will be briefly considered here.

IIA. BASIC RESONATOR EQUATIONS

For this discussion the Fox-Li Integral Equations (FLIE) will be compared with another integral equation formulation which we call the Source Integral Equation (SIE). For resonator theory the FLIE has been very useful and makes up in convenience and familiarity what it may lack in mathematical consistency. For some time, under support for AFOSR and AFWL, we have investigated the SIE, which is mathematically consistent, but not as tractible as the FLIE. In the large Fresnel number limit, the two equations predict the same output beam profiles: the Hermite-gaussians and Laguerre-gaussians. For smaller Fresnel number the predictions differ.

To proceed simply, let us consider a model using scalar optics and empty (passive) cavities with spherical mirrors of circular aperature in symmetrical, unstable configurations. In addition, a number of frequently used simplifying approximations, such as paraxial light and distant, nearly flat mirrors, will be presumed. In the steps below, the reader is invited to settle for glimpses of the "appearance" of the equations, in lieu of algebra, derivations, mathematically clear explanations, and proper explanation, even, of the notation.

Using two-dimensional vectors $\vec{p}$ and $\vec{q}$ for transverse coordinates, one can put the two equations in the form:

$$\text{(FLIE)} \qquad -\sigma\psi(\vec{p}) = \frac{i\varepsilon}{\lambda}\iint_M \frac{e^{ik\rho_2}}{\rho_2}\,\psi(\vec{q})\,d^2\vec{q}$$

$$\text{(SIE)} \qquad -\frac{i}{\lambda}\iint_M \frac{e^{ik\rho_1}}{\rho_1}\,D(\vec{q})d^2\vec{q} = \frac{i\varepsilon}{\lambda}\iint_M \frac{e^{ik\rho_2}}{\rho_2}\,D(\vec{q})d^2\vec{q} \;. \tag{1}$$

In these equations:

ψ is the Fox-Li spatial amplitude (at the actual mirror surface) such that $|\psi|^2$ is interpreted as proportional to the output at the mirror;

D is the "source density" (current) on the actual mirror surface, proportional to the output at the mirror;

σ is an ad hoc eigenvalue, introduced as an effort toward "self-consistency," interpreted as a loss factor such that $1-|\sigma|^2$ is the "fractional energy loss per transit";

$\varepsilon = \pm 1$ is a symmetry factor, positive for modes symmetric about the resonator transverse mid-plane and negative for antisymmetric modes;

$\lambda = \frac{2\pi}{k}$ is the wavelength of the monochromatic output;

M is the flat, circuilar disc projection of the actual mirror onto a a transverse plane;

ρ_2 is the distance from a point on one mirror to a point on the other, and ρ_1 is the distance to a point on the same mirror.

Features of interest which may be observed from the appearance of Eqs. (1) are:

1) the FLIE is an integral equation of the second kind which can be solved by iteration, whereas the SIE is an integral equation of the first kind, less tractible;
2) the dependent variables ψ and D do not have the same physical meaning or units, but are related so that each can represent the shape of the output beam profile;
3) because the SIE is mathematically consistent, no ad hoc self-consistency factor σ is present;
4) the notions of "transits" and "loss per transit" are not germain (or even defined) in the SIE approach, which is a boundary value approach, not an initial value approach as in the FLIE.

Replacing $\vec{p}$ and $\vec{q}$ by the dimensionless variables $\vec{u}$ and $\vec{v}$, normalized by the aperture radius, and removing the same phase factor (quadratic in $|\vec{q}|$) from both ψ and D, one can obtain the following approximate versions of Eq. (1):

$$\text{(FLIE)} \quad -\sigma f_F(\vec{v}) = i\varepsilon N e^{ikL} \iint_M e^{i\pi MN(\vec{u}-\frac{\vec{v}}{M})^2} f_F(\vec{u})\, d^2\vec{u}$$

$$\text{(SIE)} \quad \frac{a}{i\lambda} \iint_M \frac{e^{ika[|\vec{v}-\vec{u}| + B(u^2-v^2)]}}{|\vec{u}-\vec{v}|} f_S(\vec{u})\, d^2\vec{u} \; . \approx \qquad (2)$$

$$\approx i\varepsilon N e^{ikL} \iint_M e^{i\pi MN(\vec{u}-\frac{\vec{v}}{M})^2} f_S(\vec{u})\, d^2\vec{u}$$

where a is the aperture radius, L is the mirror vertex separation, N is the Fresnel number, M is the magnification,

$$B \equiv \frac{a}{2L} \sqrt{g^2 - 1}\,, \qquad g \equiv 1 - \frac{L}{R} = 1 + \frac{L}{|R|}\,,$$

and R is the (negative) mirror radius of curvature.

Because we wish to compare the solutions and, in particular, the resonance conditions of Eqs. (2), a method of solution is required. For this purpose, I will sketch a functional equation method (FEM) that is useful near the geometrical optics limit of large Fresnel number. As stated above, the two equations have the same Laguerre- gaussian solutions in this limit. It is necessary for us to presume that we are close enough to this limit that the solutions f_F and f_S resemble each other to the extent that, for modes f_F with slowly varying profile shape, the corresponding modes f_S will have slowly varying profile shape.

Now, from the appearance of the FLIE in Eqs. (2), one can observe that it is manifestly self-consistent to presume that f_F is a slowly varying function. The same observation, however, cannot be made directly from the appearance of of the SIE. Thus, f_S can be presumed slowly varying only by appealing to its similarity to f_F mentioned above.

Now, for large N the kernels in Eqs. (2) allow a reasonably slowly varying function f to contribute only near the point of stationary phase. By specifically studying the nature of the kernels and employing stationary phase and Taylor expansion techniques, one can approximate Eqs. (2) by reducing them to functional equations:

$$\begin{aligned} &\text{(FLIE/FEM)} \qquad -\sigma_n F_F(\vec{v}) \simeq G\left(\frac{\vec{v}}{M}\right) F_F\left(\frac{\vec{v}}{M}\right) \\ &\text{(SIE/FEM)} \qquad H(\vec{v}) F_S(\vec{v}) \simeq G\left(\frac{\vec{v}}{M}\right) F_S\left(\frac{\vec{v}}{M}\right) \end{aligned} \tag{3}$$

For Eqs. (3), the behavior at the singular point ($\vec{v} = 0$) of the functional equations has been extracted:

$$f(\vec{v}) \equiv |\vec{v}|^n F(\vec{v})$$

where n is an arbitrary integer. The normalization $F_F(0) = 1$, $F_S(0) = 1$ has been selected. The functions G and H are defined:

$$G(\vec{v}) \equiv \frac{iN\varepsilon e^{ikL}}{M^n} \iint_M e^{i\pi MN(\vec{u} - \vec{v})^2} d^2\vec{u}$$

$$H(\vec{v}) \equiv \frac{a}{i\lambda} \iint_M \frac{e^{ika[|\vec{u}-\vec{v}| + B(|\vec{u}|^2-|\vec{v}|^2)]}}{|\vec{u} - \vec{v}|} d^2\vec{u} .$$

Once convenient analytical expressions have been obtained, giving simple and accurate approximations for G and H, the functional equations can be easily handled and solved by iteration. This functional equation method (FEM), thus puts the FLIE and the SIE on a similar footing. Both can now be solved by an iteration procedure and, in either case, the computational effort is dramatically less than that involved for the original integral equation.

It should be clear that Eqs. (3) cannot be used at $M = 1$; i.e., for flat mirrors, because the dependent variable drops out, leaving inconsistencies. The derivation has presumed $M > 1$ and the importance of neglected higher order terms increases as $M \to 1$.

The solutions of the functional equations have been compared with solutions of the FLIE for cases with $N \approx 10$, $M \approx 3$. Although the mode shapes will not be exhibited here, the following observations were made. The FLIE/FEM mode profiles, agreed very well with the FLIE mode profiles, confirming the utility of the FEM. The SIE/FEM modes showed great similarity to those for the FLIE, but exhibited a high frequency ripple of small amplitude riding on the overall shape. This perhaps is the general nature of the SIE corrections to the FLIE, but this supposition is tentative at present.

IIB. LASER RESONANCE CONDITIONS

Now that Eqs. (3) are before us, we are finally able to discuss the resonance conditions. Eqs. (3) cannot have solutions unless they are consistent at $\vec{v} = 0$. Since $F(0) = 1$, it must, therefore, be true that:

$$\text{(FLIE)} \qquad -\sigma_n \approx G(0)$$

$$\text{(SIE)} \qquad H(0) \approx G(0) \tag{4}$$

The difference in these two resonance conditions is dramatic. In the case of the FLIE, the known function G(0) of the resonator parameters simply defines σ_n and is in itself not a condition or restriction. For the SIE, on the other hand, two definite known functions of the resonator parameters must be equal or solution is impossible. Since H and G are complex functions, each of the conditions in Eqs. (4) is actually two requirements, one for the real part and one for the imaginary part.

The FLIE condition in Eq. (4) is converted to a resonance condition in the following way. In the philosophy of the Fox-Li approach, solutions are considered to be physical only if $\sigma \equiv |\sigma| e^{i\delta}$ is such that

FLIE $$\delta(L, a, N, M) = m\pi \qquad (5)$$

for some integer m. Now, from Eq. (4) and the definition of G above, it can be shown that

$$-\sigma_n = \frac{2i\varepsilon e^{ikL}}{M^{n+1}} e^{i\frac{\pi MN}{2}} \sin\frac{\pi MN}{2} .$$

Obtaining the phase of σ_n from this expression and applying the ad hoc condition shown in Eq. (5), one finds the approximate FLIE resonance condition:

FLIE $$L \approx \frac{\lambda}{2} \left[m + \frac{1}{2} - \frac{NM}{2}\right] . \qquad (6)$$

Even though Eq. (6) has a simple appearance, this is deceptive because M and N are complicated functions of L. The resonance conditions for L are easily determined numerically, however, by iterating Eq. (6), because convergence is very fast. The resonance values are spaced at a separation of approximately one-half of a wavelength. Table I shows some of the values of m and L, around L = 100 cm for a wavelength $\lambda = 10.6\mu$, the wavelength of a CO_2 laser. The radius of curvature, R is taken to be -150 cm and the aperture radius, a, is taken to be 1 cm.

Since the Fox-Li resonance condition applies a constraint only to δ, $|\sigma|$ is left at what ever value G(0) takes on at resonance. Thus, the fraction of the energy lost per transit for the n^{th} mode is given by

$$1 - |\sigma_n|^2 \simeq 1 - [\frac{2}{M^{n+1}} \sin\frac{\pi MN}{2}]^2 . \qquad (7)$$

These approximate results obtained from the lowest order FEM may be compared with accurate computer solutions of the FLIE obtained by Siegman[1]. For this comparison, the approximate eigenvalue for the fundamental mode (n = 0) for M = 5 is shown in Fig. 1 along with the corresponding values obtianed by Siegman. The results are shown as a function of the quantity N_{eq} defined by Siegman:

$$N_{eq} \equiv \frac{N}{2} (M - \frac{1}{M}). \qquad (8)$$

The locations of the peaks and valleys agree reasonably well for the two curves.

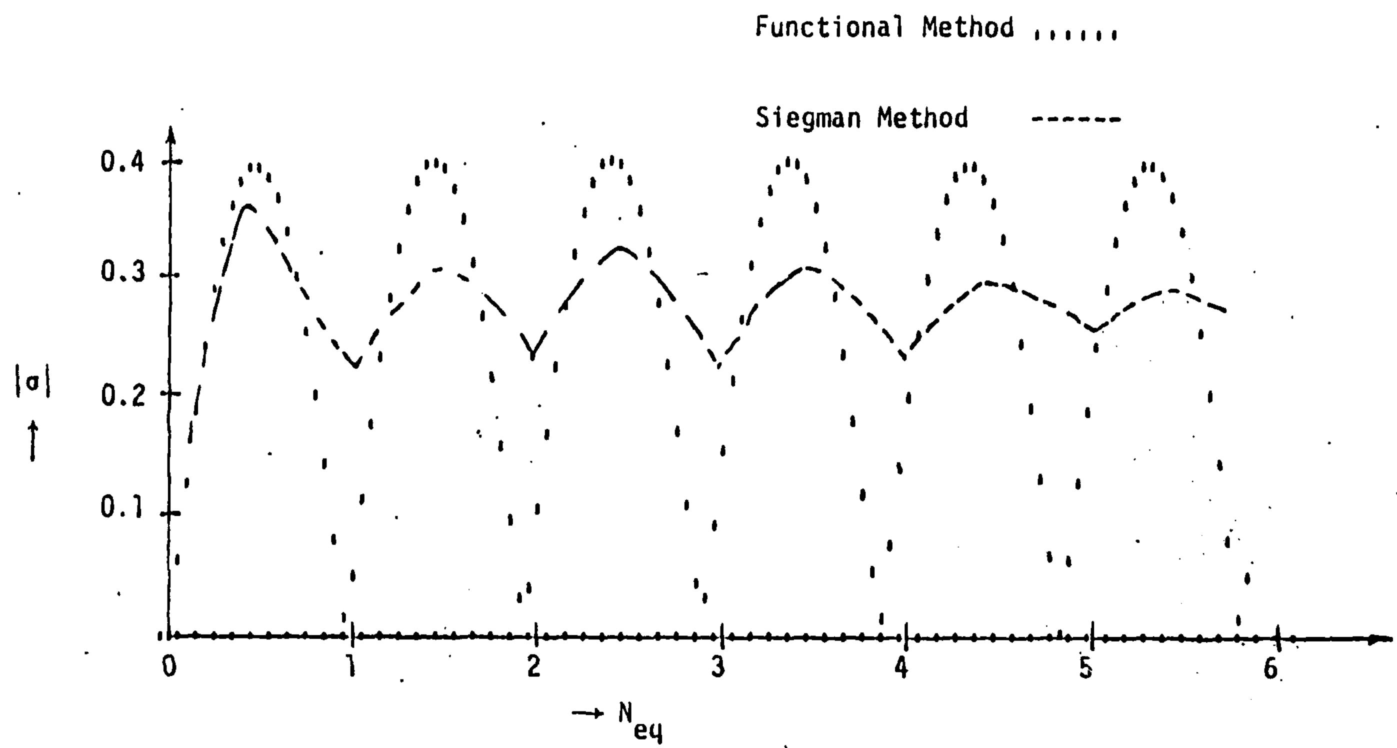
In order to study the SIE resonance condition of Eq. (4), the definitions of G and H given above are used. It can be shown that the resonance condition can be put in the form:

$$1 - \frac{e^{ika(1 + B)}}{1 + 2B} \simeq \frac{-\epsilon e^{ikL}}{M^{n+1}} (1 - e^{i\pi MN}) . \qquad (9)$$

TABLE I

Resonance Conditions for the FLIE

$a = 1$ cm,		$R = -150$ cm,	$\lambda = 10.6\ \mu = 0.00106$ cm
$L = (L_0+L_1)\ \frac{\lambda}{2}$		,	$m = m_0 + m_1$
$L_0 = 188{,}665.348535$		,	$m_0 = 188680$
m	m_1	L (cm)	L_1
188680	0	99.9926347240	0
188681	1	99.9931647436	1.000037
188682	2	99.9936947638	2.000075
188683	3	99.9942247834	3.000112
188684	4	99.9947548035	4.000150
188685	5	99.9952848231	5.000187
......		....	
188690	10	99.9979349282	10.000375
......		....	
188695	15	100.0005850210	15.000560
......		....	
188700	20	100.0032351210	20.000749
......		....	
188710	30	100.0085353200	30.001125
......		....	
188720	40	100.0138355180	40.001498
......		....	
188730	50	100.0191357170	50.001874



$M = 5$ $\qquad N_{eq} = \frac{N}{2}\left(M - \frac{1}{M}\right)$

Fig. 1 'FLIE Eigenvalue VS. N_{eq} for fixed M

To provide geometrical insight, Eq. (8) is rearranged to obtain:

$$\frac{e^{i\frac{2\pi a}{\lambda}(1+B)}}{1 + 2B} = 1 - [\frac{2\varepsilon}{M} e^{i[\frac{2\pi L}{\lambda} + \frac{\pi}{2} + \frac{\pi MN}{2}]}] \sin \frac{\pi}{2}MN$$

where, for simplicity, only the fundamental mode (n = 0) is considered. The left-hand side of Eq. (9), when plotted in the complex plane, produces a circular spiral with center at the origin and a radius of $1/(1 + 2B)$. For fixed values of a, when L increases, B decreases since $B = a\sqrt{g^2-1}\,/2L$. Thus, the spiral moves clockwise in the direction of increasing radius of spiral as illustrated in Fig. 2. Similarly, the vector on the right hand side will scan the whole circular disk with center at (1, 0) and radius bounded by 2/M. Since M increases as L increases, the radius will decrease with the increase of L and the vector will move rapidly in the counter-clockwise direction. The region scanned by the RHS vector is shaded in Fig. 2. The resonance condition is satisfied when the LHS vector and the RHS vectors meet, as illustrated by the thick arrows in the figure. The area shaded by crossed lines indicates the region in which the resonance condition can be satisfied. Keeping in mind that these vectors will also move if a is varied, one finds, for approximately fixed value of a, the resonance conditions for L to be spaced by approximately one-half of a wavelength (as was predicted by the FLIE resonance condition). This is due to the fact the LHS is a very slowly varying function of L compared to the RHS which will make approximately a full circle when L is changed by a wavelength. The resonance condition may be satisfied twice in a change of L by one wavelength, due to the two possible values, $\varepsilon = \pm 1$, for modes symmetric and antisymmetric about the resonator midplane. The numerical values of a and L, satisfying the resonance condition, are obtained by solving the simultaneous equations mentioned above.

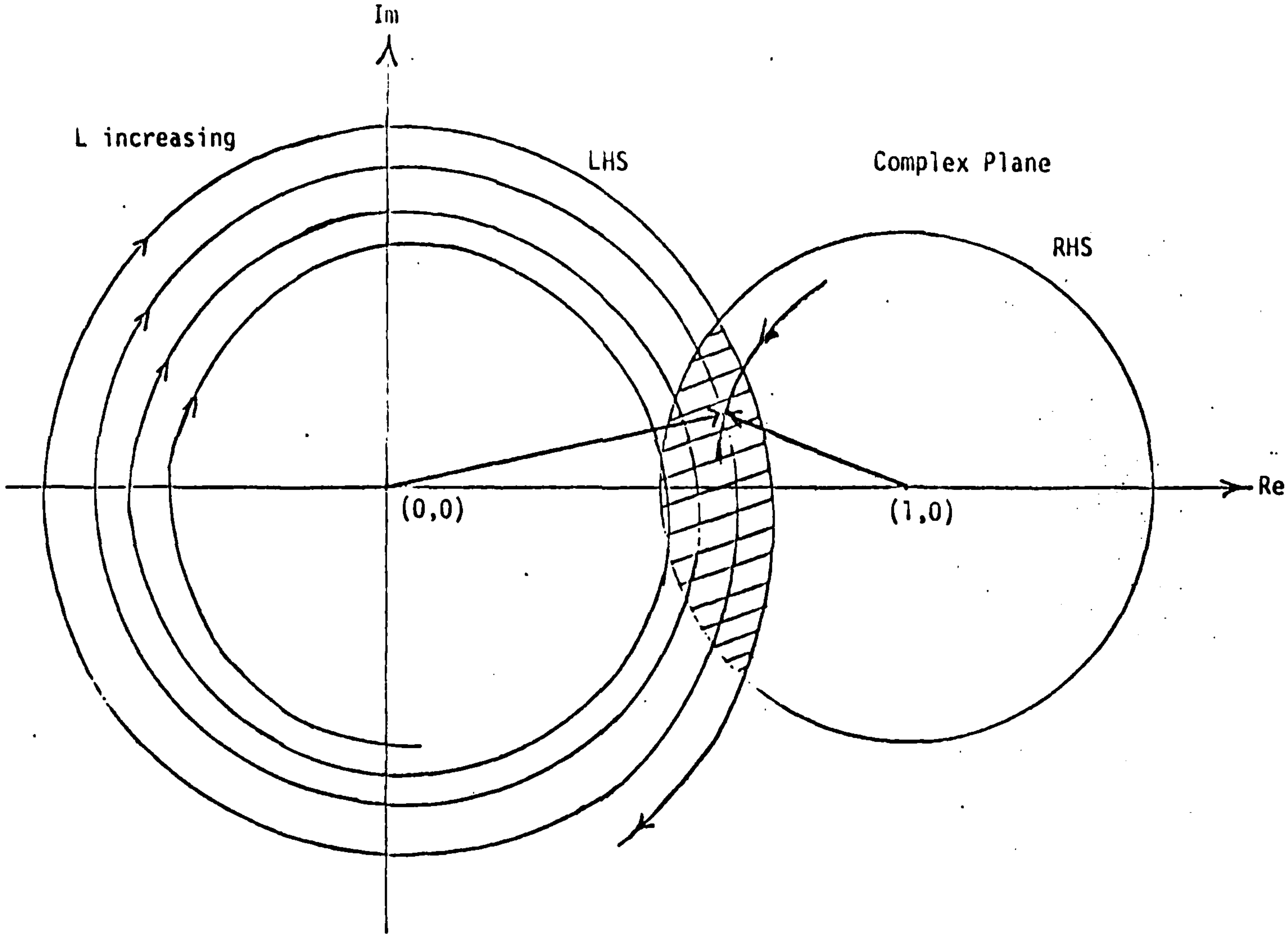


Fig. 2 Geometric Interpretation for the Resonance Conditions

In Table II a sampling of ten resonance points (a, L) is shown, taken from a list of about eighty consecutive points which were determined. For this list, a was held essentially constant and the spacing in L of consecutive resonances was essentially $\lambda/2$. To obtain resonance it was necessary to increment a by 0.000000021 cm as L increased by 0.000530021 cm. The wavelength used was $\lambda = 10.6\mu = 0.00106$ cm.

Similarly, in Table III a sampling of resonance points determined for nearly constant L is given. The spacing in a of consecutive resonances was approximately λ.

To understand clearly the differences between the FLIE and SIE resonance conditions,one can show that for both the FLIE and the SIE the resonance conditions are essentially governed by the relation:

$$\begin{matrix}\text{(FLIE)} \\ \text{(SIE)}\end{matrix} \qquad L \approx \left(m + \frac{1}{2} - \frac{NM}{2}\right)\frac{\lambda}{2} \text{ to order } \frac{a}{L} \qquad (10)$$

[One should recall that N and M also depend on L.] However, as shown earlier, this relation is the sole resonance condition for the FLIE, but such is not the case for the SIE. For the SIE there is a second condition that must be satisfied:

$$\text{(SIE)} \qquad \left|\sin\left[\frac{\pi}{2}\left(\frac{2a}{\lambda} + N_{eq}\right)\right]\right| = \frac{1}{M}\left|\sin\left(\frac{\pi}{2}\ NM\right)\right| \qquad (11)$$

The requirement that both Eq. (10) and Eq. (11) be satisfied simultaneously means that, for fixed λ and R, both L and a must be determined to satisfy the "double resonance condition".

For fixed values of λ and R, continuous curves L(a) are defined by Eq. (10) which satisfy the FLIE resonance condition. For the SIE, on the other hand, only discrete points (L, a) are allowed.

TABLE II

Resonance conditions for the SIE

	$R = -150$ cm ,	$\lambda = 10.6\ \mu$	
	$a = (a_0 + a_1)\lambda$,	$L = (L_0 + L_1)\frac{\lambda}{2}$	
	$a_0 = 933.96255$,	$L_0 = 192{,}545.35825$	
a (cm)	a_1	L (cm)	L_1
.990000318	0	102.0490398725	0
.990000339	.0000198	102.0495698946	1.0000396
.990000359	.00003868	102.0500999169	2.00008577
.990000380	.0000585	102.0506299390	3.00012547
.990000400	.00007735	102.0511599610	4.000167
....	....	....	....
.990000502	.0001736	102.0538100710	9.0003745
....	....	....	....
.990000543	.0002122	102.0548701150	11.00045755
....	....	....	....
.990000583	.00025	102.0559301590	13.00054056
....	......	....	....
.990000624	.0002887	102.056990203	15.00062358
....	....	....	....
....	....	....	....
.990003978	.00345283	102.144443839	180.007484

TABLE III

Resonance Conditions for the SIE

	$R = -150$ cm ,	$\lambda = 10.6\ \mu$	
	$a = (a_0 + a_1)\ \lambda$,	$L = (L_0 + L_1)\ \frac{\lambda}{2}$	
	$a_0 = 933.9626415$,	$L_0 = 192{,}549.358416$	
a(cm)	a_1	L (cm)	L_1
.990000400	0	102.051159961	0
.991039253	0.9800500	102.051141470	-0.0348868
.992078025	1.9600236	102.051122790	-0.07013208
.993115974	2.9392207	102.051104025	-0.1055377
.994153439	3.9179613	102.051085313	-0.14084340
.995190486	4.8963075	102.051066882	-0.17561888
.996227186	5.8743264	102.051049220	-0.20894340

In principle, this difference has some impact on the design of a resonator. If one designs from the Fox-Li viewpoint, any three of the parameters λ, a, R, L can be arbitrarily selected and the fourth determined from Eq. (10). On the other hand, design from the SIE viewpoint allows arbitrary selection of only two of the four parameters and the other two must be determined from Eqs. (10) and (11). In addition, there is a fifth parameter, $|\sigma|$, in the Fox-Li approach, conceptually related to the output coupling.

Since it is believed that the mathematically consistent SIE approach should be more accurate than the FLIE approach, one might expect that there would be some possible benefit in the SIE design approach. There is no experimental data available at present to allow assessment of this contention.

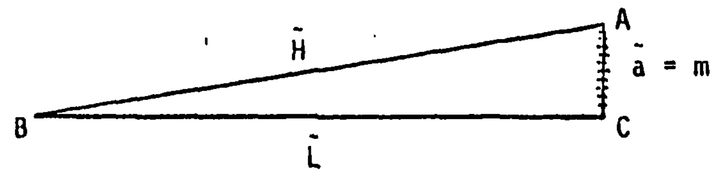
As a final brief note in this discussion, let us simply present without any derivation the lowest order SIE/FEM resonance conditions for flat circular mirrors. Imagine a right triangle whose legs are the mirror separation and the aperture radius .

$$L = \tilde{L}\lambda \qquad \text{and} \qquad a = \tilde{a}\lambda$$

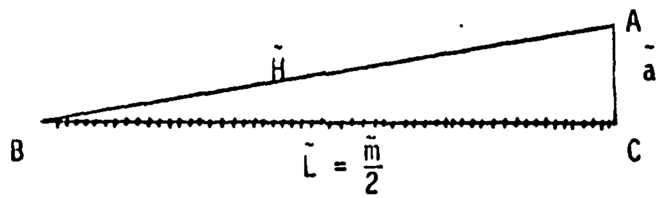
and call the hypotenuse (rim to vertex)

$$H \equiv \tilde{H}\,\lambda.$$

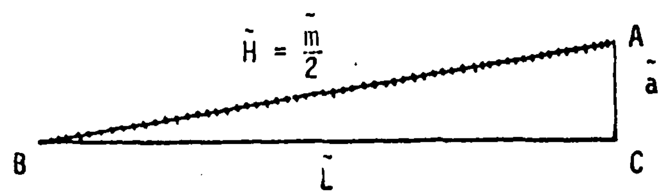
For flat mirrors there are three sets of points (L, a) which produce resonance for fixed λ and R. These sets can be summarized rather elegantly by means of the triangles shown in Fig. 3. For each set, one of the three sides is "optically short circuited" and the other two legs are of "equal optical length". Some care is required because of the availability of half-integer wavelength to $\tilde{H}$ and $\tilde{L}$ due to resonator symmetry. The full description of three sets is given by:



(a) Set I



(b) Set II



(c) Set III

Fig. 3 Physical Interpretation of Permitted Values of a, L and H

Set I. $\tilde{a} = m, \qquad \tilde{H} = \tilde{L} + n$

Set II. $\tilde{L} = \frac{\tilde{m}}{2}, \qquad \tilde{H} = \tilde{a} + \frac{\tilde{n}}{2}, \qquad \varepsilon = (-)^{\tilde{n}+1} = (-)^{\tilde{m}+1}$

Set III. $\tilde{H} = \frac{\tilde{m}}{2}, \qquad \tilde{L} = a + \frac{\tilde{n}}{2}, \qquad \varepsilon = (-)^{\tilde{n}+1} = (-)^{\tilde{m}+1}$

Note that m and n are arbitrary integers, but the integers $\tilde{m}$ and $\tilde{n}$ are correlated with the symmetry indicator (mode parity) ε.

Although the expressions above are interesting and offer insight, they are not in suitable form for numerical computation of resonance conditions. For such purposes, it is useful to note that

$$\tilde{H} \equiv \sqrt{\tilde{L}^2 + \tilde{a}^2} \approx L[1 + \frac{\tilde{a}^2}{2\tilde{L}^2}] = \tilde{L} + \frac{N}{2}$$

Using this expression to eliminate $\tilde{H}$ from the descriptions above, reveals the true complexity of the resonance conditions. For each of the sets, numerical computation is extremely simple and, in fact, one can solve analytically for L and a in terms of the integer generators m and n (or $\tilde{m}$ and $\tilde{n}$ as the case may be). Such expressions, however, appear to offer less insight than that presented in Fig. 3.

In summary, we have used a functional equation approximation method to compare resonance conditions for the FLIE and the mathematically consistent SIE. The SIE resonance conditions indicate that the role of the transverse resonator parameter a is misrepresented by the FLIE.

REFERENCE

1. Siegman, A. E., and Miller, H. Y., "Unstable Resonator Loss Calculations Using the Prony Method," Applied Optics, 9, Dec. 1970, pp. 2729-2736.

III. POSSIBLE FUTURE WORK

The work at Texas Tech under U. S. Air Force Grants No. AFOSR-73-2451 and AFOSR-77-3278 has been very productive on all four topics as evidenced by the eleven reports (850 pages). More or less by mutual agreement (or misunderstanding) this work was interrupted on March 31, 1979 and has not continued. Instead of a formal proposal for continuation, Texas Tech submitted, instead, a request for guidance and for evaluation of the relevance of the work to the Air Force Mission. Because the AFOSR grants at Texas Tech has been in effect for six and one-half years and because of substantial turn over in Air Force personnel, it appeared to be appropriate to check signals between Texas Tech, AFOSR, and the Air Force Weapons Lab. Texas Tech wished to determine if redirection of the work was necessary in order to enhance its contribution to the Air Force Mission. To date the requested assessment by the Air Force has not been completed.

A gap or hiatus in the work does not mean that Texas Tech considers results produced under this grant to be unimportant. To the contrary, results such as those discussed in Section II are <u>new</u> and potentially have great impact on and application to Air Force laser weapon design and analysis. Texas Tech will propose formal continuation of this work after appropriate response has been received from the Air Force evaluation.